Syllabus for Ph.D. Pre-Registration Qualifying Entrance Examination

MATHEMATICS

UNIT –1

Analysis:

Elementary Set theory, finite, countable and uncountable sets, Real number system as a complete ordered field, Archimedean property, supremum, infimum. Sequences and series, convergence, limsup, liminf. Bolzano weierstrass theorem, Heine Borel theorem. Continuity, uniform continuity, differentiability, mean value theorem. Sequences and series of functions, uniform convergence. Riemann sums and Riemann integral, Improper Integrals. Monotonic functions, types of discontinuity, functions of bounded variation, Lebesgue measure, Lebesgue integral. Functions of several variables, directional derivative, partial derivative, derivative as a linear transformation, inverse and implicit function theorems. Metric spaces, compactness, connectedness. Normed linear Spaces. Spaces of continuous functions.

Complex Analysis:

Algebra of complex numbers, the complex plane, polynomials, power series, transcendental functions such as exponential trigonometric and hyperbolic functions. Analytic functions, Cauchy – Riemann equations. Contour integral, cauchy's theorem, cauchy's integral formula, Liouville's theorem, Maximum modulus principle, Schwarz lemma, Open Mapping theorem. Taylor series, Calculus of residues. Conformal mappings, Mobius transformations

UNIT-2

Linear Algebra:

Vector spaces subspaces, linear dependence, basis, dimension, algebra of linear transformations. Algebra of matrices, rank and determinant of matrices, linear equations. Eigen values and eigenvectors, cayley-Hamilton theorem. Matrix representation of linear transformations. Change of basis, canonical forms, diagonal forms, triangular forms, Jordan forms, Inner product spaces orthonormal basis. Quadratic forms, reduction and classification of quadratic forms.

Algebra :

Permutations, combinations, pigeon-hole principle, inclusion-exclusion principle, derangements. Fundamental theorem of arithmetic, divisibility in Z, congruences, Chinese remainder Theorem, Euler's Ø –function, primitive roots. Groups, subgroups, normal sub groups, quotient groups, homeomorphisms, cyclic groups, permutation groups, cayley's theorems, class equations Sylow theorems. Rings, Ideals, Prime and maximal ideals, quotient rings, unique factorization domain, principal ideal domain, Euclidean domain. Polynomial rings and irreducibility criteria. Fields, finite fields, fields extensions, Galois Theory.

UNIT- 3

Ordinary Differential Equations (ODEs):

Existence and uniqueness of solutions of initial value problems for first order ordinary differential equations, Singular solutions of first order ODE's system of first order ODE's. General theory of homogenous and non-homogenous linear ODEs, variation of parameters, Sturm-Liouville boundary value problem, Green's function.

Partial Differential Equations (PDEs):

Lagrange and Charpit methods for solving first order PDEs, Cauchy problem for first order PDEs. Classification of second order PDEs, General solution of higher order PDEs with constant coefficients, Method of separation of variable for Laplace, Heat and Wave equations.

Numerical Analysis:

Numerical solution of algebraic equations, Method of iteration and Newton-Raphson method, Rate of convergence, Solution of systems of linear algebraic equations using Gauss elimination and Gauss-Seidel methods, Finite differences, Lagrange, Hermite and spline interpolation, Numerical differentiation and integration, Numerical solution of ODEs using Picard, Euler, modified Euler and Runge-Kutta methods.

Classical Mechanics:

Generalized coordinates, Lagrange's equations, Hamilton's canonical equations, Hamilton's principle and principle of least action, Two-dimensional motion of rigid bodies, Euler's dynamical equations for the motion of a rigid body about an axis, theory of small oscillations.

Calculus of Variations:

Variation of a functional, Euler-Lagrange equation, Necessary and sufficient conditions for extrema. Variational methods for boundary value problems in ordinary and partial differential equations.

UNIT – 4

Descriptive Statistics, Exploratory Data Analysis:

Sample space, discrete probability, independent events, Bayes theorem. Random variables and distribution functions (univariate and multivariate); expectation and moments. Independent random variables, marginal and conditional distributions. Characteristic functions. Probability inequalities (Tchebyshef, Markov, Jensen). Modes of convergence, weak and strong laws of large numbers, Central Limit theorems (i.i.d case).

Markov chains with finite and countable state space, classification of states, limiting behavior of n-step transition probabilities, stationary distribution, Poisson and birth-and-death processes. Standard discrete and continuous univariate distributions. Sampling distributions, standard errors and asymptotic distributions, distribution of order statistics and range. Methods of estimation, properties of estimators, confidence intervals. Tests of hypotheses: most powerful and uniformly most powerful tests, likelihood ratio tests. Analysis of discrete data and chi-square test of goodness of fit. Large sample tests, Simple nonparametric tests for one and two sample problems.

Operations Research

Linear programming problem, simplex methods, duality. Elementary queuing and inventory models. Steady-state solutions of Markovian queuing models: M/M/1, M/M/1 with limited waiting space, M/M/C, M/M/C with limited waiting space, M/G/1.

UNIT -5

Topology:

Bases interior, closure, subspaces, product spaces, continuous functions, compact spaces, locally compact spaces, separation axioms- T1, T2, T3, T4 spaces, Uryshon's lemma, connected spaces components.

Functional Analysis:

Normed spaces, quotient spaces, Banach spaces, Continuous liner transformations, dual spaces, reflexive spaces, Hahn Banoch Theorem, Open Mapping theorem, closed graph theorem, inner product spaces, Hilbert spaces, Inner product spaces, Hilbert spaces, orthonormal sets, orthogonal complement, self adjoint operators, normal operators, unitary operators, projections.

Linear Integral Equations:

Linear integral equation of the first and second kind of Fredholm and Volterra type, Solutions with separable kernels. Characteristic numbers and eigen functions, resolvent kernel.